

# Module 4

## Single-phase AC circuits

# Lesson 16

## Solution of Current in AC Parallel and Series- parallel Circuits

In the last lesson, the following points were described:

1. How to compute the total impedance/admittance in series/parallel circuits?
2. How to solve for the current(s) in series/parallel circuits, fed from single phase ac supply, and then draw complete phasor diagram?
3. How to find the power consumed in the circuit and also the different components, and the power factor (lag/lead)?

In this lesson, the computation of impedance/admittance in parallel and series-parallel circuits, fed from single phase ac supply, is presented. Then, the currents, both in magnitude and phase, are calculated. The process of drawing complete phasor diagram is described. The computation of total power and also power consumed in the different components, along with power factor, is explained. Some examples, of both parallel and series-parallel circuits, are presented in detail.

**Keywords:** Parallel and series-parallel circuits, impedance, admittance, power, power factor.

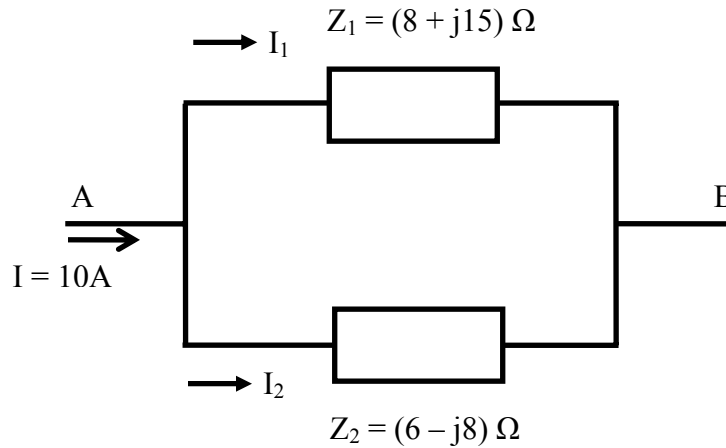
After going through this lesson, the students will be able to answer the following questions;

1. How to compute the impedance/admittance, of the parallel and series-parallel circuits, fed from single phase ac supply?
2. How to compute the different currents and also voltage drops in the components, both in magnitude and phase, of the circuit?
3. How to draw the complete phasor diagram, showing the currents and voltage drops?
4. How to compute the total power and also power consumed in the different components, along with power factor?

This lesson starts with two examples of parallel circuits fed from single phase ac supply. The first example is presented in detail. The students are advised to study the two cases of parallel circuits given in the previous lesson.

### **Example 16.1**

The circuit, having two impedances of  $Z_1 = (8 + j15) \Omega$  and  $Z_2 = (6 - j8) \Omega$  in parallel, is connected to a single phase ac supply (Fig. 16.1a), and the current drawn is 10 A. Find each branch current, both in magnitude and phase, and also the supply voltage.



**Fig. 16.1 (a) Circuit diagram**

**Solution**

$$Z_1 \angle \phi_1 = (8 + j15) = 17 \angle 61.93^\circ \Omega \quad Z_2 \angle -\phi_2 = (6 - j8) = 10 \angle -53.13^\circ \Omega$$

$$I \angle 0^\circ (OC) = 10 \angle 0^\circ = (10 + j0) A$$

The admittances, using impedances in rectangular form, are,

$$Y_1 \angle -\phi_1 = \frac{1}{Z_1 \angle \phi_1} = \frac{1}{8 + j15} = \frac{8 - j15}{8^2 + 15^2} = \frac{8 - j15}{289} = (27.68 - j51.9) \cdot 10^{-3} \Omega^{-1}$$

$$Y_2 \angle \phi_2 = \frac{1}{Z_2 \angle -\phi_2} = \frac{1}{6 - j8} = \frac{6 + j8}{6^2 + 8^2} = \frac{6 + j8}{100} = (60.0 + j80.0) \cdot 10^{-3} \Omega^{-1}$$

Alternatively, using impedances in polar form, the admittances are,

$$Y_1 \angle -\phi_1 = \frac{1}{Z_1 \angle \phi_1} = \frac{1}{17.0 \angle 61.93^\circ} = 0.05882 \angle -61.93^\circ$$

$$= (27.68 - j51.9) \cdot 10^{-3} \Omega^{-1}$$

$$Y_2 \angle \phi_2 = \frac{1}{Z_2 \angle -\phi_2} = \frac{1}{10.0 \angle -53.13^\circ} = 0.1 \angle 53.13^\circ = (60.0 + j80.0) \cdot 10^{-3} \Omega^{-1}$$

The total admittance is,

$$Y \angle \phi = Y_1 + Y_2 = [(27.68 - j51.9) + (60.0 + j80.0)] \cdot 10^{-3} = (87.68 + j28.1) \cdot 10^{-3} \\ = 92.07 \cdot 10^{-3} \angle 17.77^\circ \Omega^{-1}$$

The total impedance is,

$$Z \angle -\phi = \frac{1}{Y \angle \phi} = \frac{1}{92.07 \cdot 10^{-3} \angle 17.77^\circ} = 10.86 \angle -17.77^\circ = (10.343 - j3.315) \Omega$$

The supply voltage is

$$V \angle -\phi (V_{AB}) = I \angle 0^\circ \cdot Z \angle -\phi = (10 \times 10.86) \angle -17.77^\circ = 108.6 \angle -17.77^\circ V \\ = (103.43 - j33.15) V$$

The branch currents are,

$$I_1 \angle -\theta_1 (OD) = \frac{V \angle -\phi}{Z_1 \angle \phi_1} = \left( \frac{108.6}{17.0} \right) \angle -(17.77^\circ + 61.93^\circ) = 6.39 \angle -79.7^\circ A$$

$$= (1.143 - j 6.286) \text{ A}$$

$$I_2 \angle \theta_2 (OE) = I \angle 0^\circ - I_1 \angle -\theta_1 (OC - OD = OC - CE)$$

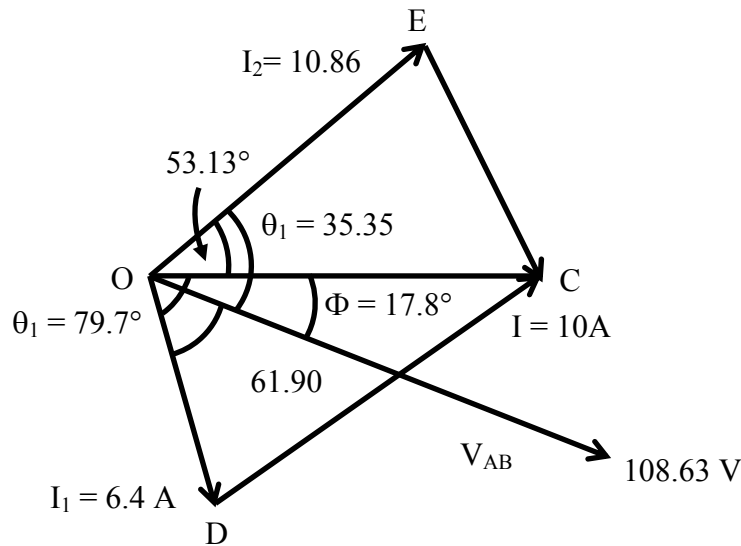
$$= (10.0 + j 0.0) - (1.143 - j 6.286) = (8.857 + j 6.286) \text{ A} = 10.86 \angle 35.36^\circ \text{ A}$$

Alternatively, the current  $I_2$  is,

$$I_2 \angle \theta_2 (OE) = \frac{V \angle -\phi}{Z_2 \angle -\phi_2} = \left( \frac{108.6}{10.0} \right) \angle (-17.77^\circ + 53.13^\circ) = 10.86 \angle 35.36^\circ \text{ A}$$

$$= (8.857 + j 6.285) \text{ A}$$

The phasor diagram with the total (input) current as reference is shown in Fig. 16.1b.



**Fig. 16.1 (b) Phasor diagram**

### Alternative Method

$$Z' \angle \phi' = Z_1 \angle \phi_1 + Z_2 \angle -\phi_2 = (8 + j 15) + (6 - j 8) = (14 + j 7) = 15.65 \angle 26.565^\circ \Omega$$

$$Z \angle -\phi = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} = \frac{Z_1 \angle \phi_1 \cdot Z_2 \angle -\phi_2}{Z' \angle \phi'} = \left( \frac{17.0 \times 10.0}{15.65} \right) \angle (61.93^\circ - 53.13^\circ - 26.565^\circ)$$

$$= 10.86 \angle -17.77^\circ = (10.343 - j 3.315) \Omega$$

The supply voltage is

$$V \angle -\phi (V_{AB}) = I \cdot Z = (10 \times 10.86) \angle -17.77^\circ = 108.6 \angle -17.77^\circ \text{ V}$$

$$= (103.43 - j 33.15) \text{ V}$$

The branch currents are,

$$I_1 \angle -\theta_1 (OD) = I \frac{Z_2}{Z_1 + Z_2} = \left( \frac{10.0 \times 10.0}{15.65} \right) \angle (-53.13^\circ - 26.565^\circ) = 6.39 \angle -79.7^\circ \text{ A}$$

$$= (1.142 - j 6.286) \text{ A}$$

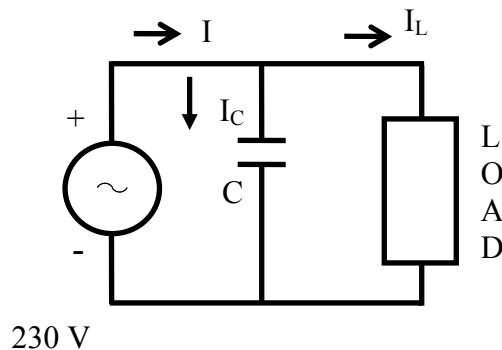
$$I_2 \angle \theta_2 (OE) = I - I_1 (OC - OD = OC - CE) = (10.0 + j0.0) - (1.142 - j6.286) \\ = (8.858 + j6.286) A = 10.862 \angle 35.36^\circ A$$

Alternatively, the current  $I_2$  is,

$$I_2 \angle \theta_2 (OE) = I \frac{Z_1}{Z_1 + Z_2} = \left( \frac{10.0 \times 17.0}{15.65} \right) \angle (61.93^\circ - 26.565^\circ) = 10.86 \angle 35.36^\circ A \\ = (8.858 + j6.286) A$$

### Example 16.2

The power consumed in the inductive load (Fig. 16.2a) is 2.5 kW at 0.71 lagging power factor (pf). The input voltage is 230 V, 50 Hz. Find the value of the capacitor C, such that the resultant power factor of the input current is 0.866 lagging.



**Fig. 16.2 (a) Circuit diagram**

### Solution

$$P = 2.5 \text{ KW} = 2.5 \cdot 10^3 = 2500 \text{ W} \quad V = 230 \text{ V} \quad f = 50 \text{ Hz}$$

The power factor in the inductive branch is  $\cos \phi_L = 0.71$  (lag)

The phase angle is  $\phi_L = \cos^{-1}(0.71) = 44.77^\circ \approx 45^\circ$

$$P = V \cdot I_L \cos \phi_L = 230 \cdot (I_L \cos \phi_L) = 2500$$

$$I_L = \frac{P}{V \cos \phi_L} = \frac{2500}{230 \times 0.71} = 15.31 \text{ A}$$

$$I_L \cos \phi_L = 15.31 \times 0.71 = 10.87 \text{ A} ; \quad I_L \sin \phi_L = 15.31 \times \sin 45^\circ = 10.87 \text{ A}$$

The current  $I_L$  is,  $I_L \angle -\phi_L = 15.31 \angle -45^\circ = (10.87 - j10.87) A$

The power consumed in the circuit remains same, as the capacitor does not consume any power, but the reactive power in the circuit changes. The active component of the total current remains same as computed earlier.

$$I \cos \phi = I_L \cos \phi_L = 10.87 \text{ A}$$

The power factor of the current is  $\cos \phi = 0.866$  (lag)

The phase angle is  $\phi = \cos^{-1}(0.866) = 30^\circ$

The magnitude of the current is  $I = 10.87 / 0.866 = 12.55 \text{ A}$

The current is  $I \angle -\phi = 12.55 \angle -30^\circ = (10.87 - j6.276) A$

The current in the capacitor is

$$I_C \angle 90^\circ = I \angle -\phi - I_L \angle -\phi_L = (10.87 - j6.276) - (10.87 - j10.87) \\ = j4.504 = 4.504 \angle 90^\circ \text{ A}$$

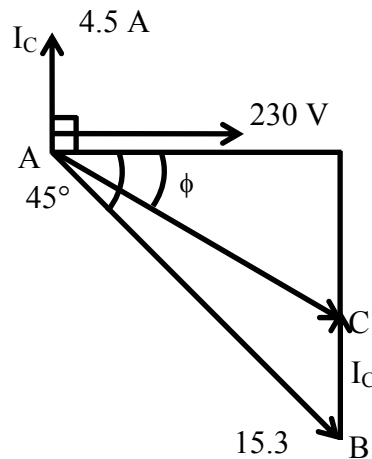
This current is the difference of two reactive currents,

$$-I_C = I \sin \phi - I_L \sin \phi_L = 6.276 - 10.87 = -4.504 \text{ A}$$

The reactance of the capacitor, C is  $X_C = \frac{1}{2\pi f C} = \frac{V}{I_C} = \frac{230}{4.504} = 51.066 \Omega$

The capacitor, C is  $C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 51.066} = 62.33 \cdot 10^{-6} = 62.33 \mu\text{F}$

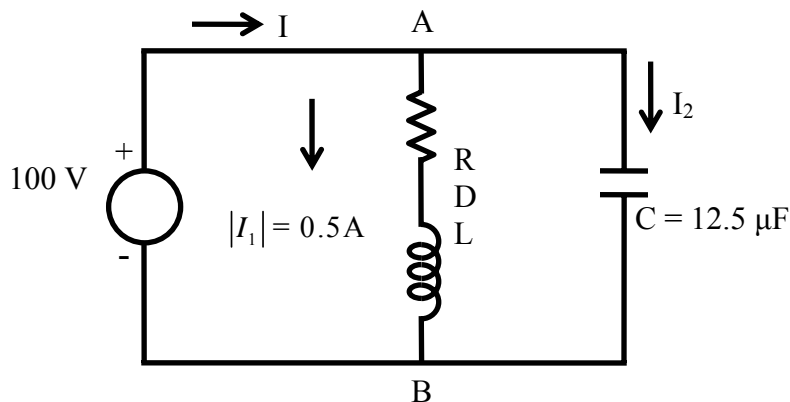
The phasor diagram with the input voltage as reference is shown in Fig. 16.2b.



**Fig. 16.2 (b) Phasor diagram**

### Example 16.3

An inductive load (R in series with L) is connected in parallel with a capacitance C of  $12.5 \mu\text{F}$  (Fig. 16.3a). The input voltage to the circuit is 100 V at 31.8 Hz. The phase angle between the two branch currents, ( $I_1 = I_L$ ) and ( $I_2 = I_C$ ) is  $120^\circ$ , and the current in the first branch is  $I_1 = I_L = 0.5 \text{ A}$ . Find the total current, and also the values of R & L.



**Fig. 16.3 (a) Circuit diagram**

### Solution

$$f = 31.8 \text{ Hz} \quad \omega = 2\pi f = 2\pi \times 31.8 \approx 200 \text{ rad/s} \quad V = 100 \text{ V}$$

$$I_1 = 0.5 \text{ A} \quad C = 12.5 \mu\text{F} = 12.5 \cdot 10^{-6} \text{ F}$$

$$X_C = 1/(\omega C) = 1/(200 \times 12.5 \cdot 10^{-6}) = 400 \Omega$$

The current in the branch no. 2 is

$$I_2 \angle 90^\circ = V / (-j X_C) = 100 \angle 0^\circ / 400 \angle -90^\circ = (100/400) \angle 90^\circ = 0.25 \angle 90^\circ \\ = (0.0 + j0.25) \text{ A}$$

The current in the branch no. 1 is  $I_1 \angle -\phi_1 = 0.5 \angle -\phi_1$

The phase angle between  $I_1$  and  $I_2$  is  $90^\circ + \phi_1 = 120^\circ$

$$\text{So, } \phi_1 = 120^\circ - 90^\circ = 30^\circ$$

$$I_1 \angle -30^\circ = 0.5 \angle -30^\circ = (0.433 - j0.25) \text{ A}$$

The impedance of the branch no. 1 is,

$$Z_1 \angle \phi_1 = (R + j X_L) = V \angle 0^\circ / I_1 \angle -30^\circ = (100/0.5) \angle 30^\circ = 200 \angle 30^\circ \\ = (173.2 + j100.0) \Omega$$

$$R = 173.2 \Omega \quad X_L = \omega L = 100.0 \Omega$$

$$\text{So, } L = X_L / \omega = 100 / 200 = 0.5 \text{ H} = 500 \cdot 10^{-3} = 500 \text{ mH}$$

The total current is,

$$I \angle 0^\circ = I_1 \angle -30^\circ + I_2 \angle 90^\circ = (0.433 - j0.25) + j0.25 = (0.433 + j0.0) \\ = 0.433 \angle 0^\circ \text{ A}$$

The total impedance is,

$$Z \angle 0^\circ = (R' + j0) = V \angle 0^\circ / I \angle 0^\circ = (100/0.433) \angle 0^\circ = 231.0 \angle 0^\circ \\ = (231.0 + j0.0) \Omega$$

The current,  $I$  is in phase with the input voltage,  $V$ .

$$\text{The total admittance is } Y \angle 0^\circ = Y_1 \angle -\phi_1 + Y_2 \angle 90^\circ = (1/Z_1 \angle 30^\circ) + (1/Z_2 \angle -90^\circ)$$

$$\text{The total impedance is } Z \angle 0^\circ = (Z_1 \angle 30^\circ \cdot Z_2 \angle -90^\circ) / (Z_1 \angle 30^\circ + Z_2 \angle -90^\circ)$$

Any of the above values can be easily calculated, and then checked with those obtained earlier. The phasor diagram is drawn in Fig. 16.3b.

## Solution of Current in Series-parallel Circuit

### Series-parallel circuit

The circuit, with a branch having impedance  $Z_1$ , in series with two parallel branches having impedances,  $Z_2$  and  $Z_3$ , shown in Fig. , is connected to a single phase ac supply.

The impedance of the branch, AB is  $Z_{AB} \angle \phi_{AB} = Z_1 \angle \phi_1$

$$Y_2 \angle -\phi_2 = \frac{1}{Z_2 \angle \phi_2} ; \quad Y_3 \angle -\phi_3 = \frac{1}{Z_3 \angle \phi_3}$$



The admittance of the parallel branch, BC is

$$Y_{BC} \angle -\phi_{BC} = Y_2 \angle -\phi_2 + Y_3 \angle -\phi_3 = \frac{1}{Z_2 \angle \phi_2} + \frac{1}{Z_3 \angle \phi_3}$$

The impedance of the parallel branch, BC is

$$Z_{BC} \angle \phi_{BC} = \frac{1}{Y_{BC} \angle -\phi_{BC}} = \frac{Z_2 \angle \phi_2 + Z_3 \angle \phi_3}{Z_2 \angle \phi_2 \cdot Z_3 \angle \phi_3} = \frac{Z_2 \angle \phi_2 + Z_3 \angle \phi_3}{(Z_2 Z_3) \angle (\phi_2 + \phi_3)}$$

The total impedance of the circuit is

$$Z_{AC} \angle \phi_{AC} = Z_{AB} \angle \phi_{AB} + Z_{BC} \angle \phi_{BC} = Z_1 \angle \phi_1 + Z_{BC} \angle \phi_{BC}$$

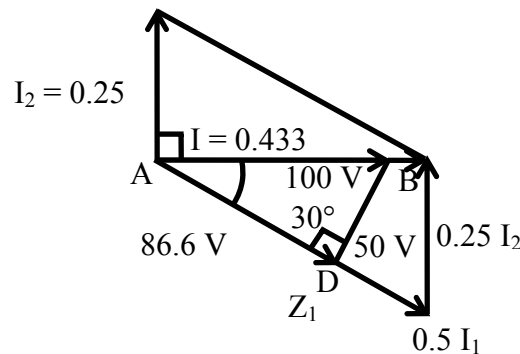
The supply current is

$$I \angle -\phi_{AC} = \frac{V \angle 0^\circ}{Z_{AC} \angle \phi_{AC}}$$

The current in the impedance  $Z_2$  is

$$I_2 \angle \phi_4 = I \angle -\phi_{AC} \frac{Z_3 \angle \phi_3}{(Z_2 \angle \phi_2 + Z_3 \angle \phi_3)}$$

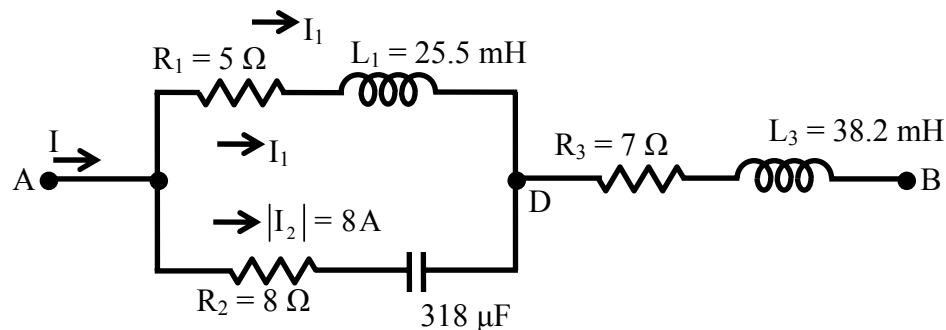
Thus, the currents, along with the voltage drops, in all branches are calculated. The phasor diagram cannot be drawn for this case now. This is best illustrated with the following examples, where the complete phasor diagram will also be drawn in each case.



**Fig. 16.3 (b) Phasor diagram**

#### Example 16.4

Find the input voltage at 50 Hz to be applied to the circuit shown in Fig. 16.4a, such that the current in the capacitor is 8 A?

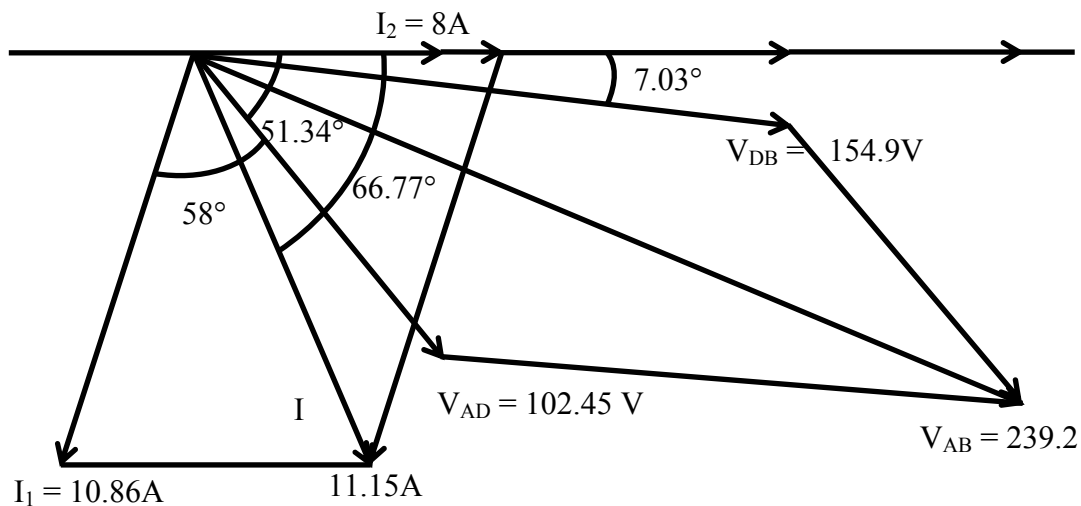


**Fig. 16.4 (a) Circuit diagram**

### Solution

$$\begin{aligned}
 f &= 50 \text{ Hz} & \omega &= 2\pi f = 2\pi \times 50 = 314.16 \text{ rad/s} \\
 L_1 &= 0.0255 \text{ H} & L_3 &= 0.0382 \text{ H} & C &= 318 \mu\text{F} = 318 \cdot 10^{-6} \text{ F} \\
 X_1 &= \omega L_1 = 314.16 \times 0.0255 = 8 \Omega & X_3 &= \omega L_3 = 314.16 \times 0.0382 = 12 \Omega \\
 X_2 &= 1/(\omega C) = 1/(314.16 \times 318 \cdot 10^{-6}) = 10 \Omega & I_2 \angle 0^\circ &= 8 \angle 0^\circ = (8 + j0) \text{ A} \\
 Z_1 \angle \phi_1 &= R_1 + jX_1 = (5 + j8) = 9.434 \angle 58^\circ \Omega \\
 Z_2 \angle -\phi_2 &= R_2 - jX_2 = (8 - j10) = 12.806 \angle -51.34^\circ \Omega \\
 Z_3 \angle \phi_3 &= R_3 + jX_3 = (7 + j12) = 13.89 \angle 59.74^\circ \Omega \\
 V_{AC} \angle -\phi_2 &= I_2 \angle 0^\circ \cdot Z_2 \angle -\phi_2 = (8.0 \times 12.806) \angle -51.34^\circ = 102.45 \angle -51.34^\circ \text{ V} \\
 &= (64 - j80) \text{ V} \\
 I_1 \angle -\theta_1 &= \frac{V_{AC} \angle -\phi_2}{Z_1 \angle \phi_1} = \left( \frac{102.45}{9.434} \right) \angle -(51.34^\circ + 58^\circ) = 10.86 \angle -109.34^\circ \text{ A} \\
 &= (-3.6 - j10.25) \text{ A} \\
 I \angle -\theta_3 &= I_1 \angle -\theta_1 + I_2 \angle 0^\circ = (-3.6 + j10.25) + (8.0 + j0.0) = (4.4 - j10.25) \\
 &= 11.154 \angle -66.77^\circ \text{ A} \\
 V_{CB} \angle -\theta_{CB} &= I \angle -\theta_3 \cdot Z_3 \angle \phi_3 = (11.154 \times 13.89) \angle (-66.77^\circ + 59.74^\circ) \\
 &= 154.93 \angle -7.03^\circ \text{ V} = (153.764 - j18.96) \text{ V} \\
 V_{AB} \angle -\theta_{AB} &= V_{AC} \angle -\phi_2 + V_{CB} \angle -\theta_{CB} = (64.0 - j80.0) + (153.764 - j18.96) \\
 &= (217.764 - j90.96) = 239.2 \angle -24.44^\circ \text{ V}
 \end{aligned}$$

The phasor diagram with the branch current,  $I_2$  as reference, is shown in Fig. 16.4b.

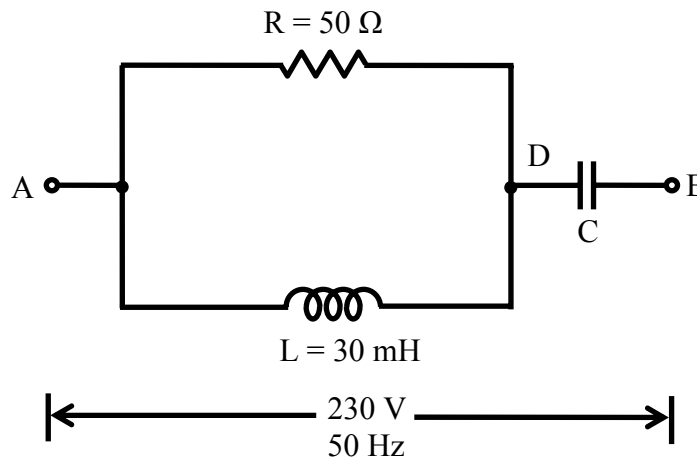


**Fig. 16.4 (b) Phasor diagram**

### Example 16.5

A resistor of  $50\ \Omega$  in parallel with an inductor of  $30\text{ mH}$ , is connected in series with a capacitor,  $C$  (Fig. 16.5a). A voltage of  $220\text{ V}$ ,  $50\text{ Hz}$  is applied to the circuit. Find,

- (a) the value of  $C$  to give unity power factor,
- (b) the total current, and
- (c) the current in the inductor



**Fig. 16.5 (a) Circuit diagram**

### Solution

$$\begin{aligned}
 f &= 50\text{ Hz} & \omega &= 2\pi f = 2\pi \times 50 = 314.16\text{ rad/s} \\
 R &= 50\ \Omega & V &= 220\text{ V} & L &= 30\text{ mH} = 30 \cdot 10^{-3} = 0.03\text{ H} \\
 X_L &= \omega L = 314.16 \times 0.03 = 94.24\ \Omega
 \end{aligned}$$

The admittance,  $Y_{AD}$  is,

$$\begin{aligned}
 Y_{AD} \angle -\phi_{AD} &= \frac{1}{R} + \frac{1}{jX_L} = \frac{1}{50} + \frac{1}{j94.24} = (20.0 - j10.61) \cdot 10^{-3} \\
 &= 0.02264 \angle -27.95^\circ\ \Omega^{-1}
 \end{aligned}$$

The impedance,  $Z_{AD}$  is,

$$\begin{aligned}
 Z_{AD} \angle \phi_{AD} &= 1/(Y_{AD} \angle \phi_{AD}) = 1/(0.02264 \angle -27.95^\circ) = 44.17 \angle 27.95^\circ \\
 &= (39.02 + j20.7)\ \Omega
 \end{aligned}$$

The impedance of the branch (DB) is  $Z_{DB} = -jX_C = -j[1/(\omega C)]$ .

As the total current is at unity power factor (upf), the total impedance,  $Z_{AB}$  is resistive only.

$$Z_{AB} \angle 0^\circ = R_{AB} + j0 = Z_{AD} \angle \phi_{AD} + Z_{DB} \angle -90^\circ = 39.02 + j(20.7 - X_C)$$

Equating the imaginary part,  $X_C = 1/(\omega C) = 20.7\ \Omega$

The value of the capacitance  $C$  is,

$$C = \frac{1}{\omega X_C} = \frac{1}{314.16 \times 20.7} = 153.8 \cdot 10^{-6} = 153.8 \mu F$$

So,  $Z_{AB} \angle 0^\circ = R_{AB} + j0 = (39.02 + j0.0) = 39.02 \angle 0^\circ \Omega$

The total current is,

$$I \angle 0^\circ = V \angle 0^\circ / Z_{AB} \angle 0^\circ = (220.0 / 39.02) \angle 0^\circ = (5.64 + j0.0) = 5.64 \angle 0^\circ A$$

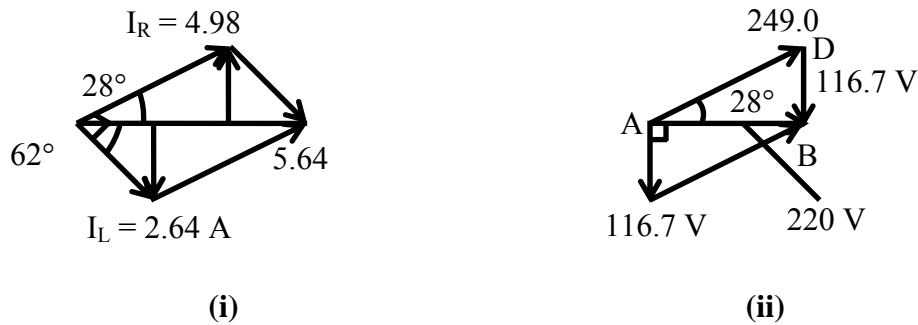
The voltage,  $V_{AD}$  is,

$$V_{AD} \angle \phi_{AD} = I \angle 0^\circ \cdot Z_{AD} \angle \phi_{AD} = (5.64 \times 44.17) \angle 27.95^\circ = 249.05 \angle 27.95^\circ V \\ = (220.0 + j116.73) V$$

The current in the inductor,  $I_L$  is,

$$I_L \angle \theta_L = V_{AD} \angle \phi_{AD} / X_L \angle 90^\circ = (249.05 / 94.24) \angle (27.95^\circ - 90^\circ) \\ = 2.64 \angle -62.05^\circ A = (1.24 - j2.335) A$$

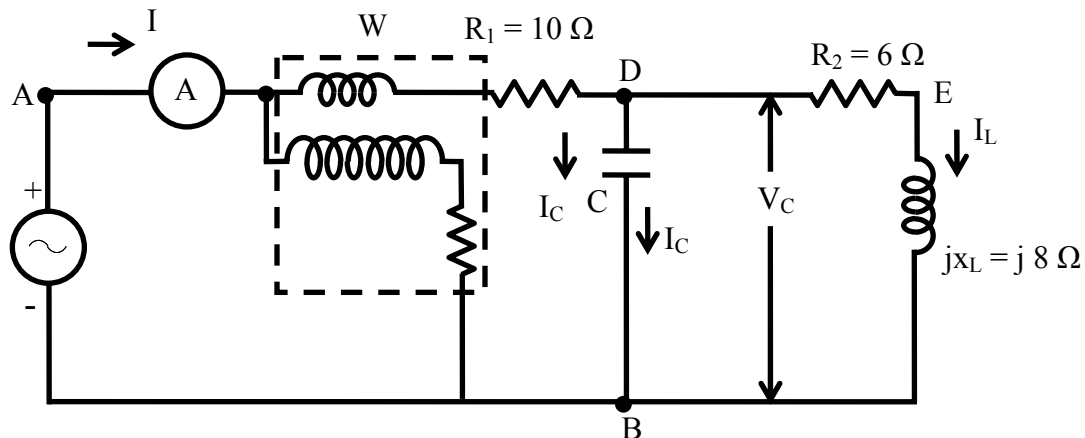
The phasor diagram is shown in Fig. 16.5b.



**Fig. 16.5 (b) Phasor diagram**

### Example 16.6

In the circuit (Fig. 16.6a) the wattmeter reads 960 W and the ammeter reads 6 A. Calculate the values of  $V_s$ ,  $V_C$ ,  $I_C$ ,  $I$ ,  $I_L$  and  $X_C$ .



**Fig. 16.6 (a) Circuit diagram**

### Solution

In this circuit, the power is consumed in two resistance,  $R_1$  and  $R_2$  only, but not consumed in inductance  $L$ , and capacitance  $C$ . These two components affect only the reactive power.

$$P = 960 \text{ W} \quad I = 6 \text{ A} \quad R_1 = 10 \Omega$$

$$R_2 = 6 \Omega \quad X_L = 8 \Omega$$

$$\text{Total power is, } P = I^2 \cdot R_1 + I_L^2 \cdot R_2 = (6)^2 \times 10 + 6 \cdot I_L^2 = 360 + 6 \cdot I_L^2 = 960 \text{ W}$$

$$\text{or, } 6 \cdot I_L^2 = 960 - 360 = 600 \text{ W}$$

$$\text{So, } I_L = \sqrt{600/6} = 10 \text{ A}$$

The impedance of the inductive branch is,

$$Z_L = R_2 + j X_L = (6 + j8) = 10 \angle 53.13^\circ \Omega$$

The magnitude of the voltage in the inductive branch is,

$$V_{DB} = V_C = I_L \cdot Z_L = 10 \times 10 = 100 \text{ V}$$

Assuming  $V_{DB} = 100 \angle 0^\circ$  as reference, the current,  $I_L$  is,

$$I_L \angle -\phi_L = V_{DB} \angle 0^\circ / Z_L \angle \phi_L = (100/10) \angle -53.13^\circ = 10 \angle -53.13^\circ \text{ A} = (6 - j8) \text{ A}$$

$$\text{The current, } I_C \angle 90^\circ = j I_C = V_{DB} \angle 0^\circ / X_C \angle -90^\circ = (V / X_C) \angle 90^\circ$$

$$\text{The total current is } I \angle \phi = I_L \angle -\phi_L + I_C \angle 90^\circ = (6 - j8) + j I_C = 6 + j(I_C - 8)$$

$$\text{So, } I = \sqrt{(6)^2 + (I_C - 8)^2} = 6 \text{ A}$$

$$\text{or, } 36 + (I_C - 8)^2 = (6)^2 = 36$$

$$\text{So, } I_C = 8 \text{ A}$$

$$\text{The capacitive reactance is, } X_C = V_{DB} / I_C = 100/8 = 12.5 \Omega$$

$$\text{The total current is } I \angle 0^\circ = (6 + j0) = 6 \angle 0^\circ \text{ A}$$

or, it can be written as,

$$I \angle 0^\circ = I_L \angle -\phi_L + I_C \angle 90^\circ = (6 - j8) + j8 = (6 + j0) = 6 \angle 0^\circ \text{ A}$$

The voltage  $V_{AD}$  is.

$$V_{AD} \angle 0^\circ = I \angle 0^\circ \cdot (R_1 + j0) = (I \cdot R_1) \angle 0^\circ = (6 \times 10) \angle 0^\circ = 60 \angle 0^\circ = (60 + j0) \text{ V}$$

The voltage,  $V_S = V_{AB}$  is.

$$V_S = V_{AB} \angle 0^\circ = V_{AD} \angle 0^\circ + V_{DB} \angle 0^\circ = (60 + 100) \angle 0^\circ = 160 \angle 0^\circ = (160 + j0) \text{ V}$$

The current,  $I$  is in phase with  $V_S = V_{AB}$ , and also  $V_{DB}$ .

The total impedance is,

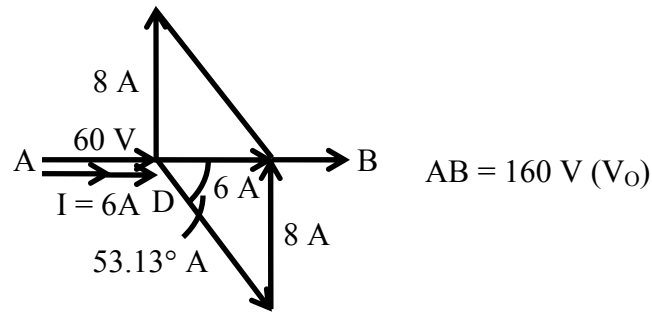
$$\begin{aligned} Z_{AB} \angle 0^\circ &= Z_{AD} \angle 0^\circ + Z_{DB} \angle 0^\circ = V_{AB} \angle 0^\circ / I \angle 0^\circ = (160/6) \angle 0^\circ = 26.67 \angle 0^\circ \\ &= (26.67 + j0.0) \Omega \end{aligned}$$

The impedance,  $Z_{DB}$  is,

$$\begin{aligned} Z_{DB} \angle 0^\circ &= Z_{AB} \angle 0^\circ - Z_{AD} \angle 0^\circ = V_{DB} \angle 0^\circ / I \angle 0^\circ = (100/6) \angle 0^\circ = 16.67 \angle 0^\circ \\ &= (16.67 + j0.0) \Omega \end{aligned}$$

Both the above impedances can be easily obtained using the circuit parameters by the method given earlier, and then checked with the above values. The impedance,  $Z_{DB}$  can

be obtained by the steps given in Example 16.3. The phasor diagram is shown in Fig. 16.6b.

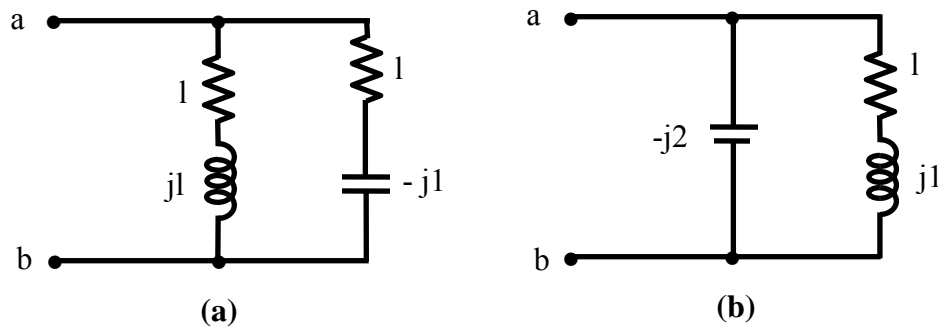


**Fig. 16.6 (b) Phasor diagram**

Starting with the examples of parallel circuits, the solution of the current in the series-parallel circuit, along with the examples, was taken up in this lesson. The problem of resonance in series and parallel circuits will be discussed in the next lesson. This will complete the module of single phase ac circuits

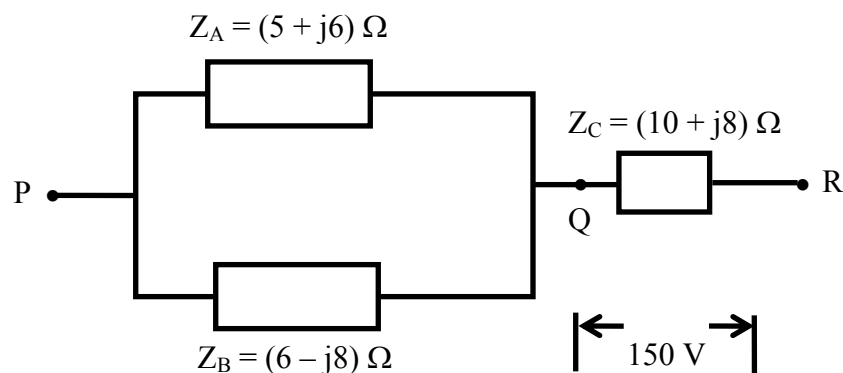
## Problems

16.1 Find the impedance,  $Z_{ab}$  in the following circuits (Fig. 16.7a-b): (check with admittance diagrams in complex plane)

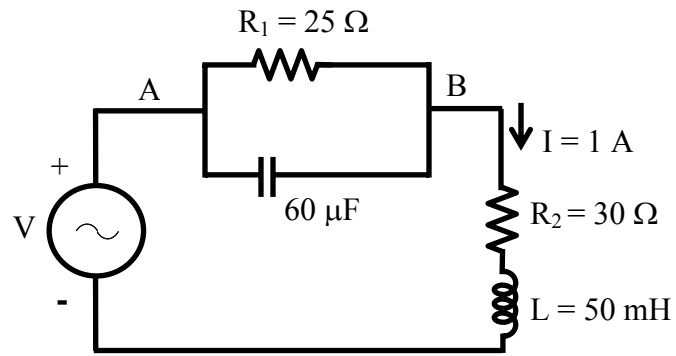


**Fig. 16.7**

- 16.2 A resistor ( $R$ ) of  $50\ \Omega$  in parallel with a capacitor ( $C$ ) of  $40\ \mu\text{F}$ , is connected in series with a pure inductor ( $L$ ) of  $30\ \text{mH}$  to a  $100\ \text{V}$ ,  $50\ \text{Hz}$  supply. Calculate the total current and also the current in the capacitor. Draw the phasor diagram.
- 16.3 In a series-parallel circuit (Fig.16.8), the two parallel branches A and B, are in series with the branch C. The impedances in  $\Omega$  are,  $Z_A = 5+j$ ,  $Z_B = 6-j8$ , and  $Z_C = 10+j8$ . The voltage across the branch, C is  $(150+j0)\ \text{V}$ . Find the branch currents,  $I_A$  and  $I_B$ , and the phase angle between them. Find also the input voltage. Draw the phasor diagram.
- 16.4 A total current of  $1\ \text{A}$  is drawn by the circuit (Fig.16.9) fed from an ac voltage,  $V$  of  $50\ \text{Hz}$ . Find the input voltage. Draw the phasor diagram.



**Fig. 16.8**



**Fig. 16.9**



## List of Figures

Fig. 16.1	(a) Circuit diagram (Ex. 16.1)	(b) Phasor diagram
Fig. 16.2	(a) Circuit diagram (Ex. 16.2)	(b) Phasor diagram
Fig. 16.3	(a) Circuit diagram (Ex. 16.3)	(b) Phasor diagram
Fig. 16.4	(a) Circuit diagram (Ex. 16.4)	(b) Phasor diagram
Fig. 16.5	(a) Circuit diagram (Ex. 16.5)	(b) Phasor diagram
Fig. 16.6	(a) Circuit diagram (Ex. 16.6)	(b) Phasor diagram